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The design of wide-band transistor feedback amplifiers

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Summary

A design technique is developed which apparently overcomes all the limitations of common-emitter transistor video amplifiers. This technique is based on the use of the impedance mismatch which occurs between stages having alternate series and shunt feedback. It is shown that the realizable gain-bandwidth product is in excess of $0 \cdot 9\,\omega_T$, the gain and bandwidth are insensitive to transistor parameter variations, and large output voltages may be obtained. The equations for both gain and bandwidth are developed in a form which is particularly suited to practical design work, and are accurate despite their comparative simplicity.

In addition to the main treatment, the design of terminal stages and of multi-stage feedback loops is considered in detail, and some aspects of the theory of noise in feedback amplifiers are discussed.

Two complete design examples are described. These are a 20 dB 25 Mc/s amplifier for 75 Ω lines using two OC170 transistors, and a vidicon head-amplifier which achieves 3×10^{-9} A noise at the input in a 5 Mc/s bandwidth.

List of symbols

$$r_E = \frac{kT}{qI_E}$$

$$r_{B'E} = \frac{r_E}{1 - \alpha_N} \simeq \beta_N r_E \rightarrow \text{looking into base}$$

(r_π)

$$g_m = \frac{\alpha_N}{r_E} \simeq \frac{1}{r_E} = \frac{\beta}{r_\pi}$$

$$r_{CB'} = \beta_N r_{CE}$$

$$\tau_1 \equiv 1/\omega_1 = r_E c_B$$

$$c_T = c_B + c_{tE}$$

$$\tau_T \equiv 1/\omega_T = r_E c_T$$

$$c_M = c_{tC}(1 - A_0)$$

$$c_C = c_B + c_{tE} + c_M$$

$$\tau_C \equiv 1/\omega_C = r_E c_C$$

$$\tau_\beta \equiv 1/\omega_\beta = \beta_N \tau_T = \beta_N \frac{c_T}{g_m} = \beta_N \frac{C_\pi}{g_m}$$

$$\frac{C_\pi}{g_m} = \frac{\overbrace{C_B + C_{tE}}^{C_\pi} + \overbrace{C_{tC}(1 - A_v)}^{C_\mu}}{g_m}$$

which here is not ω_T (which is short circuit current gain cut-off frequency, i.e. $A_v = 0$ → no Miller effect)

1 Introduction

Junction transistors have been available to circuit engineers for almost a decade. During this period, small-signal amplifier circuits have received extensive treatment in the literature. None the less, the great majority of published contributions are of little assistance in everyday amplifier design. Indeed, there seems to be a major gap between the abstract (but very necessary) contributions of network theoreticians and those of empirical or trial-and-error designers. Relatively few attempts [1–4] have been made to develop design techniques which are both simple and accurate. This contrasts with current techniques for valve-amplifier design, in which extensive use has been made of non-interacting building blocks whose dynamic performance is specified by a plot of singularities on the complex frequency plane[5, 6].

The aim of the paper is to discuss a technique for designing transistor amplifiers to satisfy the following conditions:

(a) The interaction between building blocks must be either negligible or accurately defined.
(b) The dynamic performance of individual blocks must be accurately defined.
(c) Substantial, accurately defined mid-band gain must be achieved.

The treatment is relatively simple, and although some knowledge of the simpler aspects of modern circuit theory is assumed, the paper is intended for the design engineer rather than the network specialist.

2 The small-signal model and its parameters

The design techniques developed in the paper use the hybrid-π equivalent circuit of Giacoletto[7] as the basic small-signal model of the transistor. This model is a good compromise between accuracy and complexity. Its parameters are readily measured, are directly related to device physics, have known variation laws and are quoted by some manufacturers.

The hybrid-π equivalent circuit is shown in Fig. 1. It has been shown [8, 9] that the section contained within the dotted rectangle represents the small-signal transient performance of an ideal charge-controlled device in which the output current is assumed to depend only on the instantaneous value of the total mobile charge in the base region. This charge-controlled model is neither a complete nor a wholly accurate representation of the transistor. The equivalent circuit can be completed by adding the extrinsic elements r_B, c_{tE} and c_{tC} as shown, but the inherent inaccuracies of the quasi-equilibrium (charge-control) approach remain. However, these limitations are significant only for signals of very high frequency or with rapid rates of rise. In particular, comparisons of the actual build-up of base charge with that of the charge-control model suggest[10] that the approximation is valid for complex frequencies ($s = \sigma + j\omega$) which satisfy the inequalities $|s| < \frac{1}{3}\omega'$ for diffusion transistors, and $|s| < \frac{1}{2}\omega_1$ for high-drift-field transistors.

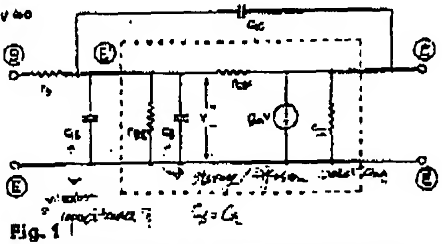

Fig. 1

The hybrid-π small-signal equivalent circuit for a transistor

The parameters are defined in Table 1

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The interrelations between the parameters of the hybrid-π equivalent circuit are given in the list of symbols. All the elements except those associated with recombination (i.e. except r_B, and hence β_N, $r_{B'E}$ and $r_{CB'}$) are dependent only on device dimensions and resistivities and are therefore reproducible with moderately close tolerances. Further, this dependence on device dimensions and resistivities accounts for a predictable variation with operating point (column 2 of Table 1) and with temperature (column 3). Lack of control in base width, minority-carrier lifetime, junction areas and base resistivity can give rise to production tolerances of the order shown in column 4, and measurements made on limited numbers of transistors of various types confirm that these tolerances are not unrealistic. Typical numerical values for the parameters of types OC 44 and OC 170 are given in column 5 of the Table. That transistors are closely reproducible in their important parameters is confirmed by the general designability of the circuits discussed in the paper.

Table 1

THE HYBRID-π EQUIVALENT-CIRCUIT PARAMETERS

(1) Equivalent circuit parameter	(2) First-order dependance on operating conditions in normal operation		(3) First-order dependence on temperature	(4) Approximate unit-to-unit tolerance referred to 'average' value	(5) Average parameters of typical types $I_E = 1$ mA, $V_C = 6$ V	
	I_E	V_C			OC44	OC170
				%		
Base resistance, r_B	—	—	$\propto T^2$	70–200	80 Ω	50 Ω
$r_{B'E}$	$\propto I_E^{-1}$	—	$\propto T$	70–200	1·5 kΩ ($\beta_N = 60$)	3·75 kΩ ($\beta_N = 150$)
r_{CE}	$\propto I_E^{-1}$	$\propto V_C^{-n}$	$\propto T$	70–140	20 kΩ	>100 kΩ
$r_{CB'}$	$\propto I_E^{-1}$	$\propto V_C^{-n}$	$\propto T$	50–300	1·2 MΩ ($\beta_N = 60$)	—
Mutual conductance, g_m	$\propto I_E$	—	$\propto T^{-1}$	98–102	40 mA/V	40 mA/V
Base-charging capacitance, c_B	$\propto I_E$	—	$\propto T^{-1}$	70–140	600 pF ($\tau_1 = 15$ ns)	40 pF ($\tau_1 = 1$ ns)
Emitter transition capacitance, c_{tE}	—	—	—	70–140	50 pF	50 pF
Collector transition capacitance, c_{tC}	—	$\propto V_C^{-n}$	—	70–140	10 pF	2 pF

The exponent n in column 2 varies with junction structure, but $0·2 \leqslant n \leqslant 0·5$

3 Limitations of non-feedback stages

The limitations of non-feedback common-emitter stages are discussed in the literature[1, 5, 6] and an acquaintance ✓ is assumed here. However, the important design techniques and results are summarized so that the advantages of the feedback approach are more apparent.

If an amplifier of the general type shown in Fig. 2 is to have a bandwidth in excess of ω_0, it is necessary that the collector supply resistance, R_C, which precedes any transistor should be much less than the input resistance of that transistor. The transistors must be fed from an approximation to a voltage source. In such circumstances, the effective load resistance, R_L, for any transistor is very nearly its own supply resistance, R_C, and the mid-band voltage gain is approximately

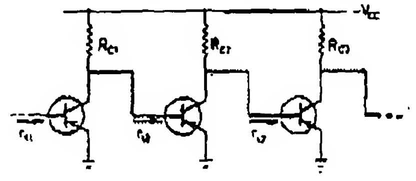

Fig. 2
Elemental circuit diagram for amplifier comprising non-feedback common-emitter stages

$$A_0 = \frac{\alpha_N}{r_B/\beta_N + r_E} R_C \quad . \quad . \quad . \quad . \quad (1)$$

At high frequencies the magnitude of the load impedance falls owing to the shunting effect of the input capacitance of the following transistor. It is possible to exchange gain for bandwidth by reducing R_C, and in ideal circumstances the gain-bandwidth product, $\mathscr{GB}$, is

$$\mathscr{GB} = \omega_C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

However, the input capacitance of a transistor is not directly accessible at a device terminal, being isolated from the base terminal by r_B. It follows that the maximum gain-bandwidth product which can be realized is

$$\mathscr{GB} = \omega_C \frac{R_C}{R_C + r_B} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and this practical value falls well below the ideal limit when the bandwidth is at all large.

Because of the transition capacitances c_{tE} and c_{tC}, it follows that the maximum gain-bandwidth product can be realized only at an optimum emitter current, $I_{E\,opt}$. This optimum current is small (for transistors such as type OC 44 in typical applications it is of the order of 100 μA); consequently, $r_{E\,opt}$ is very large compared with r_B/β_N in eqn. 1, and the mid-band gain becomes

$$A_v \simeq \frac{R_C}{r_{E\,opt}} = \frac{q}{kT} I_{E\,opt} R_C \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus, the mid-band gain is independent of β_N and is therefore predictable provided that the biasing circuits can accurately stabilize I_E. An unfortunate consequence of the small optimum emitter current is that the output voltage available from the amplifier is small.

As with valve circuits, peaking inductors may be used to increase the realizable gain-bandwidth product by up to a factor of about two per stage. The price paid is a moving of the poles off the negative real axis in the complex plane, with consequent overshoot in the transient response. /

The concept of gain-bandwidth product has little meaning when applied to terminal stages, i.e. the input and output stages of an amplifier. The realizable product depends on the source and load impedances as well as on the transistor, and values in excess of ω_C can sometimes be achieved. A low source impedance and a high load impedance favour an increase in the realizable gain-bandwidth product.

To summarize, non-feedback video amplifiers can be designed to give a stable mid-band gain but they have four disadvantages. First, the available gain-bandwidth product is considerably less than the ideal theoretical limit. Secondly, the emitter current must be highly stable if the gain is to be stable; even then the available stable gain, A_{max}, is small. It may be shown that, if the emitter current is completely stabilized and is small, so that $r_B/\beta_N \ll r_E$, the theoretical gain limit per stage for ±5% uncertainty is, with no feedback,

→ large at small current

$$A_{max} = 0 \cdot 1\,\beta_{NA} \quad . \quad . \quad . \quad . \quad . \quad (5)$$ ✓

where β_{NA} is the value of β_N expected for an 'average' transistor and the production tolerance on β_N is taken as

$$0 \cdot 7\,\beta_{NA} \leqslant \beta_N \leqslant 2\beta_{NA} \quad . \quad . \quad . \quad . \quad (6)$$

Thirdly, only a small output voltage is available before distortion becomes appreciable. Finally, the individual stages interact, and therefore a design must be started at the output stage and worked back to the input. Any change in the design of a stage necessitates a change in all preceding stages. A feedback approach to video-amplifier design overcomes these four disadvantages.

3.1 Design example

As a practical comparison between the established non-feedback design technique and the mismatched feedback approach, a simple design problem will be solved by each method. The following specification is to be met by a two-stage amplifier using type OC44 transistors:

Bandwidth	2 Mc/s
Source resistance	75 Ω
Load impedance	100 pF

The gain is to be as great as possible consistent with 5% overshoot; the same singularity pattern will be used in both the feedback and non-feedback solutions.

The elemental circuit shown in Fig. 3 gives two real and two complex poles, and one real zero. In the chosen design one of the real poles is cancelled by the zero, and the other is moved far out along the negative real axis where its effect on the pass-band is negligible; the remaining complex poles dominate and are positioned to give a 2-pole maximally flat response. The optimum emitter currents and component values may be calculated as

$$I_{E1} = 188\,\mu A \qquad R_{C1} = 275\,\Omega$$
$$I_{E2} = 144\,\mu A \qquad R_{C2} = 1 \cdot 02\,k\Omega$$
$$L = 57 \cdot 5\,\mu H$$

The average transistor parameters given in Table 1 have been assumed.

The mid-band gain of the amplifier is 11·1 and is almost independent of transistor-parameter variations. However, the bandwidth could, at the outside, be in error by as much as a factor of two. The amplifier overloads at about 70 mV r.m.s. output.

4 Introduction to mismatched feedback design

If the full benefits and simplifications of the feedback approach to video-amplifier design are to be realized, as much emphasis must be placed on the stability of mid-band gain as on bandwidth. The expressions for high-frequency gain simplify considerably if it can be assumed that the mid-band loop gain is large. The mid-band problem has been discussed elsewhere[13], but it is summarized here for completeness.

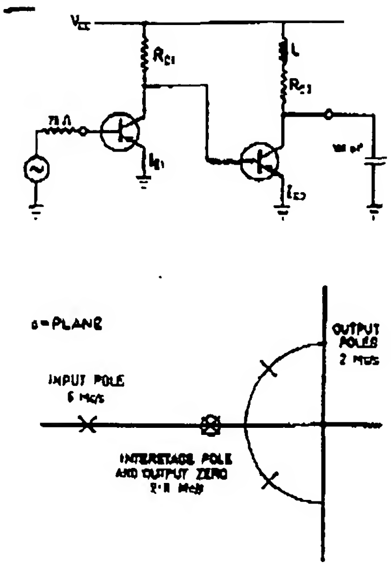

Fig. 3

Elemental circuit diagram and singularity pattern for inductively peaked amplifier

The calculated values, assuming 'average' OC44 transistors are:
R_{C1} 275 Ω R_{C2} 1·02 kΩ
L 57·5 μH
I_{E1} 188 μA I_{E2} 144 μA
Mid-band gain 11·1
Bandwidth 2·0 Mc/s
Overshoot 4·3%

4.1 Single transistor amplifier stages

There are two forms of single-stage feedback amplifier; these are the series- and shunt-feedback amplifiers shown in Fig. 4. The circuits are duals.

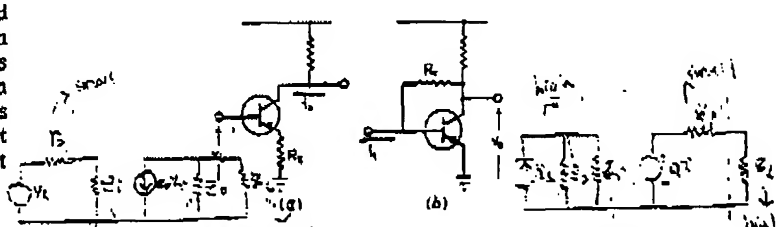

Fig. 4

Elemental mid-band circuit diagrams for two single-stage feedback amplifiers

(a) Series-feedback stage
(b) Shunt-feedback stage

for adjusting current

Series-feedback amplifier.—The series-feedback amplifier uses feedback to stabilize the transconductance, i_o/v_i. The amplifier has high input and output resistances, yet it should be terminated in a low (ideally zero) load and it must be fed from a voltage source; the feedback loop is open-circuited if a series-feedback stage is fed from a high-resistance (current) source. The stable transfer function is

low source resistance

$$\frac{i_o}{v_i} = \frac{\alpha_N}{r_B/\beta_N + r_E + R_E} \quad . \quad . \quad . \quad . \quad (7)$$

[illegible]

provided that the load resistance is not too large; this requirement will be satisfied in any normal amplifier circuit. In order to stabilize the transconductance of a series-feedback amplifier, it is necessary to make R_E large enough to mask the uncertainty in $r_B/\beta_N + r_E$, so that

the small ?
a varied Δh_{fe}

$$\frac{i_o}{v_i} \simeq \frac{1}{R_E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The principal uncertainty is in r_E; usually, the term r_B/β_N is negligibly small. The input resistance of a series-feedback amplifier is

$$r_i \simeq \beta_N R_E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whilst the output resistance is very large indeed if the amplifier is fed from a voltage source; very approximately,

$$r_o \simeq r_{CE}\frac{R_E}{r_E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Shunt-feedback amplifier.—The shunt-feedback amplifier uses feedback to stabilize the transresistance v_o/i_i. The amplifier has low input and output resistances, yet it should be terminated in a high (ideally infinite) load and it must be fed from a current source; the feedback is short-circuited to earth if a shunt-feedback amplifier is fed from a low-resistance (voltage) source. The stable transfer function is

high source resistance

$$\frac{v_o}{i_i} = -\frac{R_F}{1 + \dfrac{(R_F + r_B + \beta_N r_E)(R_F + R_L)}{\beta_N R_F R_L}} \quad . \quad (11)$$

where R_L is the total load. Usually,

$$R_L \gg)\, R_F \gg (r_B + \beta_N r_E) \quad . \quad . \quad . \quad . \quad (12)$$

so that eqn. 11 simplifies to

why ?

$$\frac{v_o}{i_i} = -\frac{R_F}{1 + (R_F + R_L)/\beta_N R_L} \quad . \quad . \quad . \quad (13)$$

In order to stabilize the transresistance of a shunt-feedback amplifier it is necessary to make R_F/R_L sufficiently small to mask the uncertainty due to β_N, so that

→ high loop gain

↳ [illegible] variation

$$\frac{v_o}{i_i} \simeq -R_F \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If the source resistance is high, as it must be for correct operation, the output resistance is

$$r_o \simeq R_F/\beta_N \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The input resistance, r_i, is given by $-R_F/A_v$ in parallel with r_1, where r_1 is the input resistance of the transistor itself and A_v is its voltage gain, given by

↳ $r_1 \simeq r_\pi = \beta r_E$

$$A_v \simeq -\frac{1}{r_B/\beta_N + r_E}\,\frac{R_F R_L}{R_F + R_L}$$

$-\frac{R_F}{A_v}$ · $\frac{R_F + R_L}{R_L}\left(\frac{r_B}{\beta} + r_E\right) \simeq \frac{r_B}{\beta_N} + r_E \ll \beta r_E$

In a satisfactory design, $-R_F/A_v$ is much smaller than r_1, so that

$$r_i \simeq (r_B/\beta_N + r_E)(1 + R_F/R_L) \quad . \quad . \quad . \quad (16)$$

dominated by $\frac{-R_F}{A_v}$

4.2 Cascades of feedback stages

There are three possible combinations for cascading feedback stages—two cascades in which all stages are of the same type, or a cascade in which the stages alternate. Owing to interaction between adjacent stages in cascades of similar stage types, the available stable gain is small. The limiting values for ±5% accuracy are, for one series stage,

$$A_{max} = 0{\cdot}1\beta_{NA} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and for one shunt stage,

$$A_{max} = 0{\cdot}1\beta_{NA} - 1 \quad . \quad . \quad . \quad . \quad . \quad (18)$$

No more gain is available than from an amplifier without feedback, but the requirement of stable emitter current is relaxed. In comparison, the theoretical limit for two adjacent stages in a cascade of alternate stages is

the following stage and the preceding in load

$$A_{max} = \beta_{NA}(0{\cdot}1\beta_{NA} - 1) \quad . \quad . \quad . \quad . \quad (19)$$

so that the advantage is about a factor of three per stage. This is so great an improvement that the alternate cascade is the only case considered at video frequencies. It is to be emphasized that eqn. 19 applies only to the stable transfer function between the input of one stage and the output of the following stage. When a series stage is followed by a shunt stage, A_{max} is the limiting voltage gain, while for a shunt stage followed by a series stage, A_{max} is the limiting current gain.

The basis of approach in the alternate cascade is to introduce a gross impedance mismatch between adjacent stages, so that there is substantially no interaction. Further, each stage operates under very nearly the ideal conditions for which its stable transfer function is defined, and the overall gain may be calculated fairly accurately by simply multiplying the individual stage transimmittances, as given by eqns. 7 or 8 and 13 or 14. Thus, when a series stage is followed by a shunt stage, the approximate voltage gain is

$$A_v \simeq \left(\frac{i_o}{v_i}\right)_1\left(\frac{v_o}{i_i}\right)_2 \simeq \frac{1}{R_{E1}}R_{F2} \quad . \quad . \quad . \quad (20)$$

while for a shunt stage followed by a series stage, the exact current gain is

$$A_i = \left(\frac{v_o}{i_i}\right)_1\left(\frac{i_o}{v_i}\right)_2 \simeq R_{F1}\frac{1}{R_{E2}} \quad . \quad . \quad . \quad (21)$$

A small correction is required in a precise calculation of the gain when a shunt stage follows a series stage. While the input resistance of a shunt stage is small, it is not zero. Therefore the output current of the series stage is not exactly equal to the input current of the shunt stage, since a small fraction of this output current is shunted by the collector supply resistance, R_{C1} in Fig. 5. The coupling efficiency η is given by

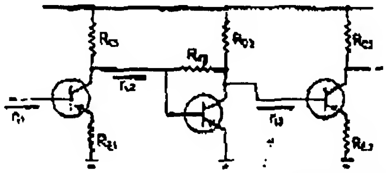

Fig. 5
Elemental mid-band circuit diagram for a cascade of alternate series- and shunt-feedback stages

→ if $r_{i3} \gg R_{E2}$, and if [illegible] stage is dominated by R_{E2}

$$\eta = \frac{R_C}{R_C + r_i} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$\frac{R_{C1}}{R_{C1} + r_{i2}}$ note $r_{i2} \ll R_{C1}$

It is rare that η is less than 0·95, so that the approximate expression for r_i, eqn. 16, is adequate even in a precise calculation of gain. It is apparent that R_C should be as large as possible. No correction is required when a series stage follows a shunt stage, since the output voltage of the shunt stage is exactly equal to the input voltage of the series stage.

There is a slight interaction between stages when a series stage follows a shunt stage, but it is significant only when the gain approaches the limiting value given by eqn. 19. The input resistance of the series stage [illegible] affects its gain slightly (eqn. 13). It is this loading effect which gives rise to the theoretical stable-gain limit. The limit is calculated on the assumption that the d.c. collector supply resistance, R_{C1} in Fig. 5, is infinite. When a finite resistance is used, the loading on the shunt stage is increased and the available stable gain is reduced. Again, it is apparent that should be as large as possible.

5 Single-stag feedback building blocks

It was shown in the preceding Section that it is possible to design cascades of alternate series- and shunt-feedback stages which are so mismatched at low frequencies that there is no interaction between stages, provided that the gain does not exceed a certain limiting value. It is not possible to produce this degree of mismatch at high frequencies, but it will be shown in this Section that a feedback video amplifier may be broken into non-interacting mismatched groups of two stages. A multi-stage feedback video amplifier is therefore more readily designable than a non-feedback amplifier. Further major advantages of the feedback approach will become apparent as the analysis proceeds.

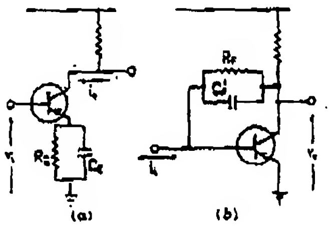

Fig. 6
Elemental circuit diagrams for single-stage feedback amplifiers with high-frequency peaking
(a) Series-feedback stage
(b) Shunt-feedback stage

Fig. 6 shows circuits for series- and shunt-feedback stages with suitable high-frequency peaking components. More elaborate inductive peaking circuits are possible, but the resulting pole-zero patterns become so complex that the circuits are not readily designable. Further, the circuits shown are capable of realizing the theoretical gain-bandwidth product limit for the transistors, so there is little point in adding further components. One important exception is the special case considered in Section 6.4.1 of a terminal stage whose load is purely resistive.

5.1 Series-feedback stage

Fig. 7(*a*) is the complete equivalent circuit for a series-feedback stage with peaking. A number of the elements have negligible effect in a practical design. Since Z_L is small in a properly designed cascade of alternate stages, $r_{CB'}$ and r_{CE} may always be omitted. For the same reason, c_{tC} may be omitted from the output circuit. The Miller component of c_C depends on the voltage gain (see list of symbols), but eqns. 7 and 16 show that the voltage gain of a series-feedback stage in a long cascade is usually less than unity, so that this Miller component is usually negligible. However, eqn. 34 gives a first-order correction. The simplified equivalent circuit is shown in Fig. 7(*b*).

If
$$\tau_E = R_E C_E \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

the transfer admittance and input impedance of a series-feedback stage with peaking are given by

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$$\frac{i_o(s)}{v_i(s)} = \frac{\alpha_N}{r_B/\beta_N + r_E + R_E} \times$$
$$\frac{1 + s\tau_E}{1 + s\left[\tau_E + \frac{r_B\tau_T + R_E(\tau_T - \tau_E)}{r_B/\beta_N + r_E + R_E}\right] + s^2\left[\frac{r_B\tau_T\tau_E}{r_B/\beta_N + r_E + R_E}\right]} \quad . \quad . \quad . \quad . \quad (24)$$ ✓

and

$$z_i(s) = \frac{v_i(s)}{i_i(s)} = r_B + \beta_N\left(\frac{r_E}{1 + s\tau_T} + \frac{R_E}{1 + s\tau_E}\right)\left(\frac{1 + s\tau_T}{1 + s\beta_N\tau_T}\right) \quad . \quad . \quad . \quad . \quad (25)$$

In the general case, the transfer admittance has two poles and one zero while the input impedance has three poles and three zeros. However, there is an important special case which gives a very great simplification, as both functions become single-poled. This simplification is achieved by putting

by adjusting $R_E C_E$ → sole purpose of C_E is introducing

$$\tau_E = \tau_T \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$ single pole ?

which gives

$$\frac{i_o(s)}{v_i(s)} = \frac{\alpha_N}{r_B/\beta_N + r_E + R_E}\;\frac{1}{1 + s\tau_T\frac{r_B}{r_B/\beta_N + r_E + R_E}} \quad . \quad (27)$$

and
$$z_i = r_B + \beta_N(r_E + R_E)\frac{1}{1 + s\beta_N\tau_T} \quad . \quad . \quad (28)$$

by and Z_L

Usually, the term r_B in eqn. 28 is negligible, even at high frequencies.

If the mid-band loop gain, A_L, is large, i.e. if

$R_E >> \frac{r_B}{\beta_N} + r_E$

$$A_L \equiv \frac{R_E}{r_B/\beta_N + r_E} > 1 \quad . \quad . \quad . \quad . \quad (29)$$

the expressions become

$$\frac{i_o(s)}{v_i(s)} \simeq \frac{1}{R_E}\,\frac{1}{1 + s\tau_T(r_B/R_E)} \quad . \quad . \quad . \quad (30)$$

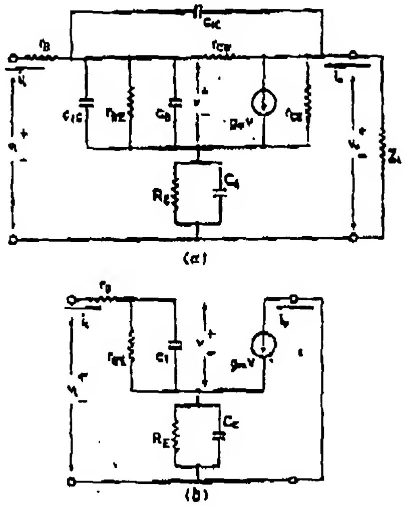


Fig. 7
Equivalent circuit for a series-feedback stage with high-frequency peaking
(a) Complete
(b) Simplified

$\tau_T = r_B C_T$ & $C_T = (C_E + C_\pi)$

and
$$z_i(s) \simeq \beta_N R_E \frac{1}{1 + s\beta_N \tau_T} \quad . \quad . \quad . \quad (31)$$

The single-poled input impedance corresponds to a parallel RC combination, in which

$$r_i = \beta_N R_E(1 + 1/A_L) \simeq \beta_N R_E \quad . \quad . \quad . \quad (32)$$

and
$$c_i = \frac{C_E}{1 + 1/A_L} \simeq C_E \quad . \quad . \quad . \quad . \quad (33)$$

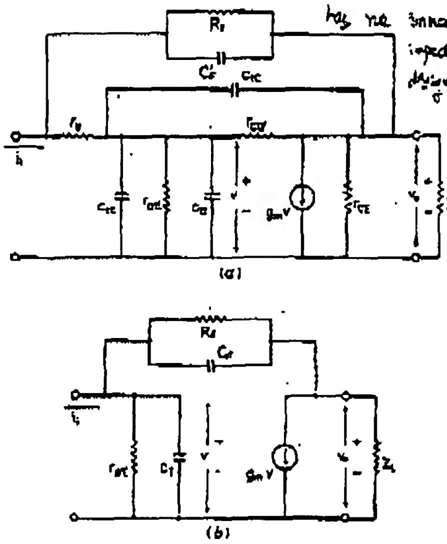
If the Miller input capacitance is taken into account,

$$c_i \simeq \frac{C_E}{1 + 1/A_L} + c_{tC}\left(1 + \frac{R_L}{R_E}\right) \quad . \quad . \quad . \quad (34)$$

The high output impedance of the series-feedback amplifier is maintained up to the cut-off frequency of the transfer admittance; the output conductance and capacitance are less than $1/r_{CE}$ and c_{tC}, respectively. Therefore it is easy to achieve $|Z_L|$ sufficiently small for eqns. 27–34 to describe the performance of the stage adequately. For all practical purposes there is no reaction of the load on the transfer admittance of a series-feedback amplifier.

5.2 Shunt-feedback stage

Fig. 8(a) is the complete equivalent circuit for a shunt-feedback stage with peaking. If the source and feedback impedances are large compared with r_B, r_B may be neglected;



— [illegible] has the immediate impact on amplifier design

Fig. 8
Equivalent circuit for a shunt-feedback stage with high-frequency peaking
(a) Complete
(b) Simplified

a factor of five is abundant and two is sufficient, so that this approximation is valid in any normal design. Further, r_{CE} and $r_{CB'}$ may be omitted in comparison with normal values for Z_L and R_F; in special circumstances r_{CE} and $r_{CB'}$ may be included in Z_L and R_F. When these simplifications are made, the equivalent circuit of Fig. 8(b) results. Note that the total feedback capacitance is

$$C_F = C_F' + c_{tC} \quad . \quad . \quad . \quad . \quad . \quad (35)$$

A number of new symbols are introduced to simplify the notation:

$$\tau_F = R_F C_F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$Z_F = R_F \frac{1}{1 + s\tau_F} \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

$$Z_P = \frac{Z_F Z_L}{Z_F + Z_L} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Subject to the condition that

$$|Z_F(s)| \gg |\beta(s) r_E| \quad . \quad . \quad . \quad . \quad . \quad (39)$$

which will be satisfied in any normal practical design (see eqn. 12), the transfer impedance of a shunt-feedback amplifier is

$$\frac{v_o(s)}{i_i(s)} = -\frac{Z_F(s)}{1 + \dfrac{Z_F(s)}{\beta(s) Z_P(s)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

$$= -\frac{R_F}{1 + s\tau_F + \dfrac{R_F(1 + s\beta_N \tau_T)}{\beta_N Z_P(s)}} \quad . \quad . \quad (41)$$

In the general case of Z_L having multiple poles and zeros, the transfer impedance becomes complicated and the stage is not readily designable. However, if Z_L is a single-poled function corresponding to an RC load, a designable two-pole transfer function results. Suppose that

→ as is the case of [illegible] follower [illegible]

$$Z_L = R_L \frac{1}{1 + sR_L C_L} \quad . \quad . \quad . \quad . \quad (42)$$

If the mid-band loop gain, A_L, is large, i.e. if

$$A_L = \frac{\beta_N R_L}{R_F + R_L} \gg 1 \quad . \quad . \quad . \quad . \quad (43)$$

the transfer impedance is given to a very good approximation by

$$\frac{v_o(s)}{i_i(s)} = -\frac{R_F}{1 + (R_F + R_L)/\beta_N R_L} \times \frac{1}{1 + s\left[R_F C_F + \dfrac{R_F + R_L}{R_L}\tau_T + \dfrac{R_F(C_F + C_L)}{\beta_N}\right] + s^2[R_F(C_F + C_L)\tau_T]} \quad . \quad . \quad . \quad . \quad (44)$$

Two further approximations are useful. First, since the loop gain has been assumed large, the denominator of the first term in eqn. 44 is approximately unity. The mid-band transfer impedance therefore reduces to R_F, the approximate value obtained in eqn. 14. A much poorer approximation is to neglect all but the first term in the coefficient of s; this approximation is useful in the early stages of a practical design. The simplified transfer impedance is

[illegible] R_L is high, [illegible] is [illegible]

$$\frac{v_o(s)}{i_i(s)} \simeq -R_F \frac{1}{1 + sR_F C_F + s^2 R_F(C_F + C_L)\tau_T} \quad . \quad (45)$$

The two poles lie on a circle whose radius is

$$|s| = 1/\sqrt{[R_F(C_F + C_L)\tau_T]} \quad . \quad . \quad . \quad (46)$$

and their real part is given approximately by

$$\sigma = -\frac{C_F}{2(C_F + C_L)\tau_T} \quad . \quad . \quad . \quad . \quad (47)$$

$$\sigma = -\frac{R_F C_F + \dfrac{R_F + R_L}{R_L}\tau_T + \dfrac{R_F(C_F + C_L)}{\beta_N}}{2R_F(C_F + C_L)\tau_T} \quad . \quad (48)$$

The requirement for a shunt-feedback stage to be designable is that its load impedance should be a single-poled RC function. For the output stage of an amplifier, this corresponds to the usual practical situation. However, within a cascade of alternate stages, the load is the input impedance of the following series-feedback stage. This is a single-poled RC function only if the series stage is peaked with $\tau_E = \tau_T$, when the input resistance and capacitance are given by eqns. 32 and 33, respectively. Therefore the condition for simplifying the design of the series-feedback stage is also the condition for designability of a long cascade.

5.3 Gain-bandwidth product of single-stage feedback building blocks

Consider an ideal cascade in which all transistors are identical. The load presented to any shunt stage is the input impedance of the following series stage in parallel with its own collector supply resistance R_C. If the loop gain around the series stage is large and if the stage is peaked with $\tau_E = \tau_T$ as is required for designability of the cascade, eqns. 32 and 33 give the load resistance and capacitance for the shunt stage as

$$R_L = \frac{r_i R_C}{r_i + R_C} \simeq \frac{\beta_N R_E R_C}{\beta_N R_E + R_C} \quad . \quad . \quad . \quad (49)$$

and
$$C_L = c_i \simeq C_E = \tau_T / R_E \quad . \quad . \quad . \quad . \quad (50)$$

In any practical case
$$C_L \gg C_F \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

so that C_F may be neglected in comparison with C_L. Substitution of these data into eqn. 44 gives the transfer impedance of the shunt stage as

$$\frac{v_o(s)}{i_i(s)} = -R_F \frac{1}{1 + sK + s^2(R_F/R_E)\tau_T^2} \quad . \quad (52)$$

where
$$K = R_F C_F + \left(\frac{R_F + R_C}{R_C} + \frac{2R_F}{\beta_N R_E}\right)\tau_T \quad . \quad . \quad (53)$$

If eqn. 52 is multiplied by the transfer admittance of the series stage, eqn. 30, the overall current gain of the two stages is

$$A_i(s) = \frac{R_F}{R_E} \frac{1}{1 + s\tau_T(r_B/R_E)} \frac{1}{1 + sK + s^2(R_F/R_E)\tau_T^2}$$

i.e.

$$A_i(s) = A_i(0) \frac{1}{1 + s\tau_T(r_B/R_E)} \frac{1}{1 + sK + s^2\tau_T^2 A_i(0)} \quad . \quad (54)$$

where
$$A_i(0) = R_F/R_E \quad . \quad . \quad . \quad . \quad . \quad (55)$$

is the mid-band current gain of the two stages.

The current gain of the two transistors is a 3-poled function. A useful simplification is to move the pole due to the series stage out along the negative real axis towards infinity by making R_E large compared with r_B. The remaining poles, due to the shunt stage, may be made coincident by putting

$$K = 2\tau_T \sqrt{A_i(0)} \quad . \quad . \quad . \quad . \quad . \quad (56)$$

and the gain of the two stages becomes

$$A_i(s) = A_i(0) \frac{1}{[1 + s\tau_T\sqrt{A_i(0)}]^2} \quad . \quad . \quad . \quad (57)$$

amplifier consists of two identical single-poled stages with individual gain-bandwidth products $\mathscr{GB}$, the gain of the amplifier is

$$A(s) = A(0) \frac{1}{[1 + s(1/\mathscr{GB})\sqrt{A(0)}]^2} \quad . \quad . \quad (58)$$

Comparison of eqns. 57 and 58 shows that the cascade of alternate stages has realized an effective gain-bandwidth product per transistor of $1/\tau_T$, the theoretical limit for an ideal transistor which has r_B reduced to zero.

In practice, the available gain-bandwidth product will be slightly less than $1/\tau_T$. The pole due to the series stage cannot be moved out indefinitely far, for, as R_E is increased, R_F must be increased to maintain the current gain and C_F must be reduced to maintain the required feedback time-constant, τ_F. A point is reached at which the required value of C_F is less than the collector transition capacitance c_{tC}, which sets an absolute minimum to C_F. A further limitation lies in the assumption that C_L is very much greater than C_F (eqn. 51). In practice, 80–90% of the theoretical gain-bandwidth product is realizable. This is better than can be obtained with most valve circuits, and is a very great improvement over non-feedback transistor circuits.

The amplifier with a double pole on the axis gives zero overshoot on transients. If an overshoot can be tolerated, the bandwidth may be increased somewhat without loss of gain. For example, if τ_F is adjusted so that

$$K = \sqrt{2}\,\tau_T \sqrt{A_i(0)} \quad . \quad . \quad . \quad . \quad . \quad (59)$$

the amplifier has a 2-pole maximally flat response which gives an increase in bandwidth of 56% with only 4·3% overshoot. This moving of poles off the negative real axis corresponds to inductive peaking of a non-feedback amplifier.

The advantages of the feedback approach to transistor video amplifier design are as follows:

(*a*) It overcomes that limitation of non-feedback designs which is due to r_B, as the realizable gain-bandwidth product is substantially ω_T.

(*b*) It ensures that the mid-band gain is accurately defined without the requirement of highly stable emitter current.

(*c*) It allows the transistors to be operated at convenient emitter currents, so that relatively large output voltages and currents may be obtained.

(*d*) Because of the almost complete independence of the transfer admittance of a series-feedback stage on its load, a multi-stage feedback amplifier may be broken up into a number of non-interacting blocks, each of which consists of a shunt stage followed by a series stage.

5.4 Accuracy of feedback approach at high frequencies with practical transistors

The theoretical analyses of the preceding Sections are based on the assumption that τ_T is known. In practice, when unselected transistors are used in amplifiers, τ_T is likely to vary between 70% and 140% of its expected value. Similar spreads occur in the other high-frequency parameters, r_B and c_{tC}. The purpose of this Section is to discuss the effects of this uncertainty in transistor parameters on the accuracy of the feedback design approach.

A series-feedback amplifier with peaking adjusted for a nominal transistor will have a zero at the expected pole frequency and two poles in the neighbourhood of this zero. Consider a stage using transistors with nominal r_B but with the production spread in τ_T. If the mid-band loop gain is five or more, the transfer admittance will be within 10% in magnitude and 10° in phase of that for a theoretical stage up to the expected pole frequency, subject, of course, to the validity of the hybrid-π equivalent circuit at such frequencies. The input impedance is no longer single-poled; in addition to the pole at $s = -1/\beta_N\tau_T$, there are an insignificant pole and zero in the neighbourhood of $-1/\tau_T$ which cancel when τ_T has its nominal value. The dominating component of the input impedance is a capacitance which is very close to the expected value given by eqn. 35.

The spread in r_B has no effect on the input impedance since r_B is neglected in eqn. 31 and thereafter. However, the frequency of the pole in the transfer admittance is almost directly proportional to r_B (eqns. 27 and 30). Experience suggests that it is good practice to design a complete amplifier so that the poles due to series stages are well out along the negative real axis, beyond the dominant poles of the overall transfer function. In this way, the uncertainty in their location due to r_B becomes insignificant. Further, the small errors due to the uncertainty of τ_T are rendered even less significant.

Finally, the uncertainty in c_{tC} is of no consequence in a series-feedback amplifier. As discussed in Section 5.1, the output capacitance and the Miller component of input capacitance, which are both due to c_{tC}, are neglected in any normal design.

The suggested minimum loop gain of five ensures that, with normal biasing circuits and the resultant degree of stability in I_E and hence r_E, the transconductance of a series-feedback amplifier is stable to better than ±5%. However, the loop gain is not so large that it may be taken as infinite in calculating the transconductance, the pole frequency and the input capacitance (eqns. 7, 30 and 33, respectively).

The expression for the transfer impedance of a shunt-feedback stage depends more on a large mid-band loop gain than the corresponding expression for a series-feedback stage. It is therefore wise to base a design on a loop gain of 10 or more. This will result in a mid-band transresistance which is stable to about ±5% with the usual tolerance on β_N.

The poles of a shunt-feedback stage lie on a circle whose radius is given by eqn. 46. The load capacitance, C_L, is known to within ±5% or better in a long cascade and is presumably known exactly when a shunt stage drives an external load. The feedback capacitance is much smaller than C_L and is usually known to within ±30%. The total capacitance, $C_F + C_L$, is therefore known to better than ±10%. If R_F is chosen to give a desired pole-circle radius with nominal component values, the uncertainty in capacitance together with the spread τ_T can give a maximum error of ±20% in the radius, with ±10% being a more likely figure.

The real part of the poles is given approximately by eqn. 47. The effect of the spread in τ_T can be taken up by making C_F a variable peaking adjustment. Since C_F includes c_{tC}, which has a spread also, the nominal value of C_F should be at least twice that of c_{tC}. Variation of the external feedback capacitance C_F' can then take up the combined spread of component values.

It appears that a feedback video amplifier is designable to ±5% in mid-band gain and 10–15% in bandwidth if the following rules are observed:

For series stage:

(a) $A_L \equiv \dfrac{R_E}{r_B/\beta_N + r_E} \geqslant 5$

(b) $C_E R_E = \tau_T$ (nominal) so that $c_t = \dfrac{C_E}{1 + 1/A_L}$

(c) Nominal pole frequency $s\left(= -\dfrac{r_B/\beta_N + r_E + R_E}{r_B\tau_T}\right)$ to be well beyond the overall cut-off frequency, by at least a factor of 2.

For shunt stage:

(d) $A_L \equiv \dfrac{\beta_N R_L}{R_F + R_L} \geqslant 10$

(e) R_F to be chosen to give required pole-circle radius, assuming nominal C_L.

(f) C_F to be adjusted to give required real part, and $C_F \geqslant 2c_{tC}$.

5.5 Numerical design

Consider the design of a feedback amplifier to meet the specification given in Section 3.1. The amplifier will consist of a series stage followed by a shunt stage, as in Fig. 9. The shunt stage will be peaked to give a 2-pole maximally flat response with the poles at 45° on a circle of radius $|s| = 12\cdot6 \times 10^6$ rad/s; their real part is $\sigma = 12\cdot6 \times 10^6 \cos 45° = -8\cdot88 \times 10^6$. The pole due to the series stage is placed on the axis at three times the overall cut-off frequency, i.e. $\sigma = -3 \times 12\cdot6 \times 10^6 = -37\cdot7 \times 10^6$.

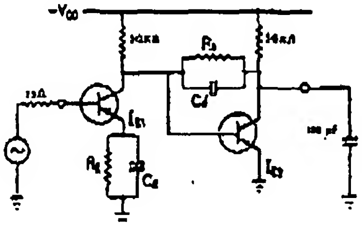


Fig. 9

Elemental circuit diagram for a two-alternate-stage feedback amplifier

The calculated values, assuming 'average' OC44 transistors are:

R_E	58·4 Ω	C_E	283 pF
R_F	3·20 kΩ	C_F'	15 pF
$I_{E1} = I_{E2} \approx$ 1 mA			
Mid-band gain	34·5		
Bandwidth	2·0 Mc/s		
Overshoot	4·3%		

The transistors will be operated at $I_E = 1$ mA for convenience.

Series stage.—Eqn. 27 gives the pole at

$$\sigma = -\frac{(r_B + R_S)/\beta_N + r_E + R_E}{r_B + R_S}\,\frac{1}{\tau_T}$$

Substitution of the numerical data gives $R_E = 58\cdot4\,\Omega$ and eqn. 26 gives $C_E = 283$ pF.

Shunt stage.—The stage will be designed with a collector supply resistance of 3·3 kΩ, a reasonable practical value. Substitution of numerical data into eqns. 46 and 48 gives $R_F = 3\cdot20$ kΩ and $C_F = 25$ pF. But $c_{tC} = 10$ pF; therefore $C_F' = C_F - c_{tC} = 15$ pF.

Gain.—The voltage gain is given by eqns. 7, 13 and 22 as

$$A_v(0) = \frac{\alpha_N}{(r_B + R_S)/\beta_N + r_E + R_E}\ \eta\ \frac{R_F}{1 + (R_F + R_L)/\beta_N R_L}$$

The value of η will depend on the collector supply resistance for the series stage, but if this is taken as 3·3 kΩ, $A_v(0) = 34\cdot5$. The amplifier overloads at about 1 V r.m.s. output.

The performance of this amplifier is much superior to that of the non-feedback design (Section 3.1).

The preceding theory of feedback building blocks can be modified for the special case of a terminal stage, i.e. the input or output stage of an amplifier.

6.1 Conditions for increased gain-bandwidth product

It has been shown that the performance of an amplifier consisting of a number of alternating shunt- and series-feedback stages is limited by the interaction between a shunt stage and the following series stage. The load capacitance for a shunt stage is the input capacitance of the following series stage, and this gives rise to the limit in gain-bandwidth product. If a shunt stage is terminated by a high load impedance or if a series stage is fed from a low source impedance, the stage gain-bandwidth product may be increased.

For a shunt stage the pole-circle radius, and hence the bandwidth, is inversely proportional to the square root of load capacitance. Consequently, if the load capacitance is smaller than the input capacitance of a series stage, the gain-bandwidth product is increased.

For a series stage, the input impedance falls as the feedback is reduced to increase gain. If the source impedance is low, the feedback can be reduced considerably without the increased input capacitance reacting on a preceding stage. For the same reason the peaking may be adjusted to give a multiple-poled transfer admittance and the resulting zero may be used to cancel a pole due to a later stage in the amplifier with a further increase in bandwidth.

Consequently, for those situations in which the source impedance is low and the load impedance high, a considerable increase in gain-bandwidth product is possible for the terminal stages. This situation is often met in practice.

The extreme case of increase in gain-bandwidth product with terminal stages is an amplifier which consists entirely of terminal stages, i.e. a two-stage amplifier. Both the numerical design examples of the previous Sections fall into this category. The outstanding example is the feedback design in which the gain-bandwidth product achieved exceeds the theoretical limit for the transistors with r_B reduced to zero. However, this represents a special case; no such extreme improvements in gain-bandwidth product are possible in general.

6.2 Choice of feedback types for terminal stages

The choice of the input and output feedback stages in a multi-stage amplifier depends on the source and load impedances. In many cases the correct choice is obvious; e.g. in the numerical example of Section 5.6, the 75 Ω source is so low that a series input stage was the obvious choice, while the 100 pF load is so high that a shunt stage was used. However, there is no sharp line of demarcation between high and low impedances and a range exists which could be regarded as either. The better type of terminal stage for impedances in this range can be found with certainty only by comparing numerical designs for the two types, but a few guiding principles apply.

First, as the emitter current is increased, some equivalent circuit impedances fall. It may be possible to change these impedances so far that a source or load of 'doubtful' class becomes obviously 'high' or 'low'. However, this technique cannot always be used as the emitter current may be constrained by other considerations, such as minimum r_T or maximum available output.

Secondly, the expression for the mid-band transfer impedance of a shunt-feedback amplifier is based (eqn. 12) on the assumption that $R_F \gg \beta_N r_E$. Further, if the mid-band gain of a shunt-feedback amplifier is to be stable to ±5%, it may be shown that

$$R_F/R_L \leqslant 0{\cdot}1\beta_{NA} - 1 \quad . \quad . \quad . \quad . \quad (60)$$

Combining these equations gives

$$R_L > \frac{\beta_{NA}}{0{\cdot}1\beta_{NA} - 1} r_E$$

or, as a working rule, assume

$$R_L \gg 3\frac{\beta_{NA}}{0{\cdot}1\beta_{NA} - 1} r_E$$

If β_{NA} is large, the smallest load resistance which should be used with a shunt stage is

$$R_{L\,min} \simeq 30 r_E \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Thirdly, the voltage gain of a series-feedback amplifier should not be large, or the expression for its transfer admittance, eqn. 27, may be inaccurate. The true value may be calculated readily provided that r_{CE}, r_{CB} and especially c_{tC} are taken into account, but eqn. 27 should be regarded as suspect if the voltage gain exceeds 3. Therefore, unless a more elaborate calculation is carried out, a guiding rule is $A_v \simeq R_L/R_E \leqslant 3$, i.e.

$$R_{L\,max} \simeq 3R_E \quad . \quad . \quad . \quad . \quad . \quad (62)$$

6.3 Comparison of feedback and non-feedback terminal stages

It was shown in the preceding Section that sources and loads of any magnitude can be used with at least one of the single-stage feedback amplifiers. Thus, the mismatched feedback design approach is applicable for all sources and loads. The purpose of this Section is to suggest that series- or shunt-feedback stages are, in general, more satisfactory as terminal stages than any other amplifier configuration.

Common-emitter stages have been discussed in Section 3, where it was stated that their gain-bandwidth product is less than the ideal limit. Common-emitter stages have no advantages over feedback stages at the input or output of an amplifier.

A common-collector (emitter-follower) input stage has a high input resistance but its input impedance has, among others, a pole at $s = -1/\beta_N \tau_T$. The input impedance therefore has a large capacitive component and $|z_i|$ does not remain large at high frequencies. A series-feedback stage can achieve an identical input impedance and is a much more versatile building block than the emitter follower. An emitter-follower output stage has no advantages over a suitable feedback stage, usually the shunt. The output impedance of an emitter follower rises with increasing frequency, since $|\beta|$ falls at 6 dB/octave:

$$z_o \simeq r_E + \frac{r_B + Z_S}{\beta(j\omega)}$$

Further, except in special circumstances, the input impedance of an emitter follower is not a single-poled function; the stage preceding an emitter follower is therefore undesignable.

A common-base stage has a low input impedance and a high output impedance. It would be possible to use a common-base input stage as an impedance transformer with unity current gain to drive a fairly large load impedance with a current and obtain a voltage output. However, a shunt feedback stage would be preferable; its low input impedance is better maintained with increasing frequency and its low output impedance would give a more stable voltage output into a varying load. A common-base output stage can achieve no higher output impedance than a series-feedback stage, while the series stage is a much more flexible and generally useful building block.

6.4 Coaxial cables

A type of terminal stage which is worthy of special mention is one which must work with a coaxial cable. In the usual wide-band case, an amplifier for use with cables will use drift transistors operating at about 5 mA for optimum performance. Unfortunately, an impedance of 50–100 Ω, typical for cables, falls into the 'doubtful' class of Section 6.2 when the emitter current is 5 mA.

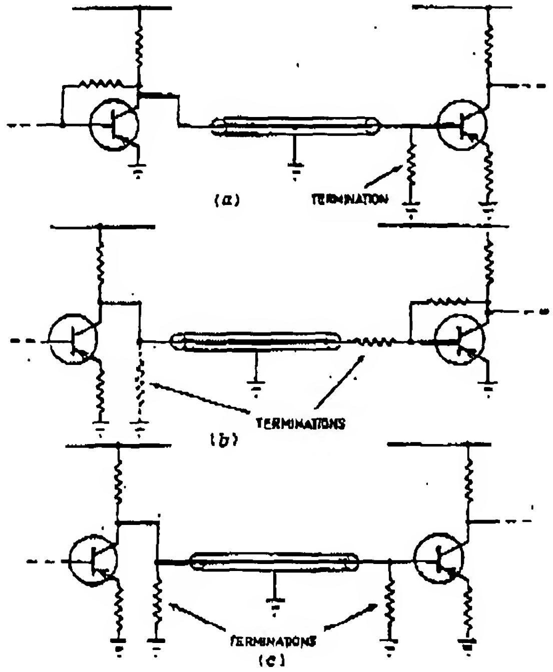


Fig. 10
Elemental circuit diagrams for three arrangements of terminal stages and coaxial cables
The terminating pads required for cable matching are shown as resistances
(a) Shunt sending-end stage, series receiving-end stage
(b) Series sending-end stage, shunt receiving-end stage
(c) Series sending-end stage, series receiving-end stage

Fig. 10 shows three possible approaches to coaxial-cable amplifier design. Note that the terminating pads, shown as resistances, may be networks involving resistance, capacitance and inductance, and should be designed to give the best cable match after allowing for the input or output impedance of the amplifier proper; a practical realization of one such network is given in Section 8.1.

Fig. 10(a) shows what is probably the most useful arrangement. Effectively, it is a cascade of a shunt stage followed by a series stage, with a rather small collector supply resistance for the shunt stage formed by the cable and its termination. The input impedance of the series-feedback amplifier at the receiving end is so high that an accurate match can be obtained from the shunt pad. No attempt is made to obtain a match at the sending end; the cable is driven from the relatively low output impedance of a shunt stage, and this output impedance is both reactive and variable. Provided that the cable length and characteristics are such that single-end termination is satisfactory, this arrangement is capable of the highest gain.

Fig. 10(b) is the dual arrangement of a series stage followed by a shunt stage. The cable is fed from a current source and terminated at the receiving end in a pad in series with the low input impedance of a shunt stage. The receiving-end match with this arrangement is unlikely to be as good as that obtainable in Fig. 10(a) because the input impedance of the shunt stage, Fig. 10(b), is not as low relative to the cable impedance as the input impedance of the series stage in Fig. 10(a) is high relative to the cable; the uncertainty in the total terminating impedance is greater because the relatively unknown input impedance of the transistor is a more significant contribution. However, if the termination shown dotted at the sending end is included, an excellent match is possible at this point. The cable then has one fair termination and one good termination, but the price paid is a 6 dB fall in gain; half the current available for the line is shunted by the sending-end pad.

Fig. 10(c) shows a circuit which gives good matches at both ends of the cable, but its gain is very low. It consists essentially of two cascaded series-feedback stages, which must give a very much lower gain than an alternate cascade. As an order of magnitude, the gain is about the same as could be obtained if one transistor and the cable were omitted.

6.5 Voltage drive for small resistive loads

One exception to the statement made in Section 5, that the inclusion of inductance in the peaking circuits is not desirable, occurs when a shunt stage is used to drive a small load which is substantially resistive up to the amplifier cut-off frequency. The most likely example of such a load is a terminated coaxial cable. Fig. 11(a) shows a practical circuit in outline, and Fig. 11(b) shows the equivalent circuit after the usual simplifications. It is assumed that R_L is so small that

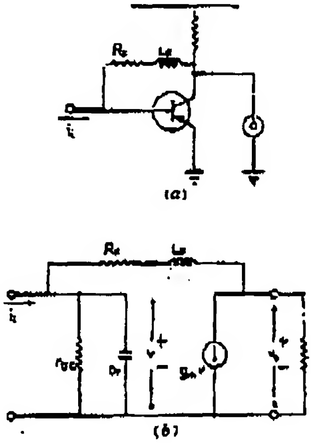


Fig. 11
Shunt-feedback stage with inductive peaking for use with small resistive loads
(a) Elemental circuit diagram
(b) Simplified equivalent circuit

$$Z_p = R_L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

at all frequencies and that c_{tC} is so small that its reactance is negligible compared with Z_F at all frequencies.

If
$$Z_F = R_F + sL_F = R_F(1 + s\tau_F) \quad . \quad . \quad . \quad (64)$$

the transfer impedance of the stage is given by eqn. 40 as $v_o/i_i = -Z_F/(1 + Z_F/\beta Z_p)$, i.e.

$$\frac{v_o(s)}{i_i(s)} = -\frac{R_F}{1 + R_F/\beta_N R_L} \times \frac{1 + s\tau_F}{1 + s\left[(\tau_F + \beta_N\tau_T)\frac{R_F}{R_F + \beta_N R_L}\right] + s^2\left[\beta_N\tau_F\tau_T\frac{R_F}{R_F + \beta_N R_L}\right]} \quad . \quad . \quad . \quad . \quad (65)$$

$$A_L \simeq \frac{\beta_N R_L}{R_F} \gg 1 \quad . \quad . \quad . \quad . \quad . \quad (66)$$

eqn. 65 reduces to

$$\frac{v_o(s)}{i_i(s)} = -\frac{R_F}{1 + R_F/\beta_N R_L} \times \frac{1 + s\tau_F}{1 + s(\tau_T + \tau_F/\beta_N)(R_F/R_L) + s^2(\tau_F\tau_T)(R_F/R_L)} \quad . \quad . \quad . \quad . \quad (67)$$

and approximating further,

$$\frac{v_o(s)}{i_i(s)} \simeq -R_F \frac{1 + s\tau_F}{1 + s\tau_T(R_F/R_L) + s^2\tau_F\tau_T(R_F/R_L)} \quad . \quad (68)$$

so that the pole-zero pattern is almost independent of β_N. The poles lie on a circle whose radius is

$$|s| = 1/\sqrt{(\tau_F\tau_T R_F/R_L)} \quad . \quad . \quad . \quad . \quad (69)$$

and have a real part given by

$$\sigma = -\frac{1}{2\tau_F} - \frac{1}{2\beta_N\tau_T} \simeq -\frac{1}{2\tau_F} \quad . \quad . \quad . \quad (70)$$

There is a zero at

$$\sigma = -1/\tau_F \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

A practical design using inductive peaking is given in Section 8.1.

7 Multi-stage feedback loops

7.1 Feedback pairs

Classical feedback theory shows that the sensitivity to parameter changes decreases as the number of stages enclosed by a feedback loop increases. Valve amplifiers are usually designed in accordance with this principle, and feedback loops around three or more stages are common. However, the variability of certain transistor parameters and the internal feedback within the transistor cause transistor feedback amplifiers to become increasingly difficult to design as the number of stages within a loop is increased. At audio frequencies a building-block of two stages is the largest which is readily designable. Two such groups, the current and voltage feedback pairs shown in Fig. 12, are particularly useful in ±10% uncertainty in gain due to the production spread in β_N, either pair is capable of a maximum gain given by

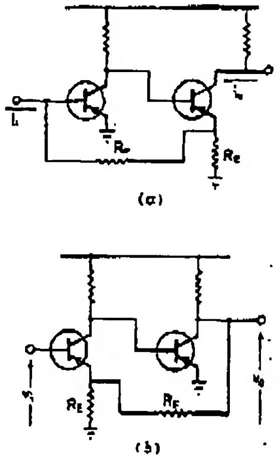


Fig. 12

Elemental mid-band circuit diagrams for feedback pairs

(a) Current feedback pair: $A_i \simeq R_F/R_E$
(b) Voltage feedback pair: $A_v \simeq R_F/R_E$

$$A_{max} = 0{\cdot}1\beta_{NA}^2 \quad . \quad . \quad . \quad . \quad . \quad (72)$$

This is slightly more than the alternate cascade, eqn. 19, but for values of β_{NA} in excess of 100 the difference is less than 10%. Audio-frequency amplifiers in which maximum stable gain is of paramount importance should therefore be designed as a cascade of feedback pairs; in other cases the greater flexibility and simpler design equations of the alternate cascade may outweigh the slightly greater gain of the feedback pairs.

The design of a feedback pair at video frequencies is tedious. The transfer function of either pair without feedback has two poles on the negative real axis in the neighbourhood of $s = -1/\tau_T\beta_N$. If the loop gain is made large enough to give a precisely stabilized gain, the closed-loop poles take up positions undesirably close to the imaginary axis and the transient response of the amplifier develops a large overshoot. It is possible to improve the transient response of a feedback pair by adding phase-correcting networks to the circuit. However, such networks are not readily designable because of the uncertainty in loop gain and open-loop pole positions due to variation of β_N. In addition, voltage-feedback pairs interact when cascaded and are therefore of little use in multi-stage video amplifiers. Current-feedback pairs do not interact in this way.

The final and most important argument against the use of feedback pairs at video frequencies is that, as shown in Section 5.3, the alternate cascade is capable of yielding the theoretical gain-bandwidth-product limit for the transistors. A feedback pair can, at the best, achieve only the same end at the expense of greater design complexity.

Ghausi[4] in a careful investigation of the current-feedback pair describes a design which uses a pair of r.f. transistors to give a two-pole maximally flat bandwidth of 2·29 Mc/s for a mid-band current gain of 20. If the same transistors are used in an amplifier consisting of a single shunt stage followed by a single series stage, the maximally flat bandwidth for a mid-band current gain of 20 is 2·26 Mc/s.

7.2 Overall feedback

A worth-while arrangement of feedback over more than one stage is the use of an overall feedback loop around a cascade of alternate series- and shunt-feedback stages. The open-loop singularities required for a given closed-loop response can be determined from standard feedback theory[4] and, since the open-loop performance can be controlled accurately by the local feedback loops, such amplifiers are readily designable.

There are at least two cases in which an overall feedback loop is advantageous. The first is when a very accurately controlled gain is required, as in a measuring amplifier. As an example, the gain of the forward path can easily be controlled to ±5 – 10% by the local loops, and a loop gain of 10 around the overall loop will result in a gain accuracy better than ±1%. The second and less obvious case is in certain low-noise applications. This is discussed in Section 7.3.

Overall feedback loops can be designed to stabilize any of the four possible transfer functions i_o/v_i, v_o/v_i, i_o/i_i and v_o/i_i; Fig. 13 shows suitable arrangements. The stable transfer function of the amplifier without the overall loop should in all cases be the function which is to be realized by the overall loop; thus, in Fig. 13(a) in which a very stable transfer conductance is to be realized, the amplifier consists of an odd number of stages with a series stage at both input and output, so that its stable transfer function is its transconductance.

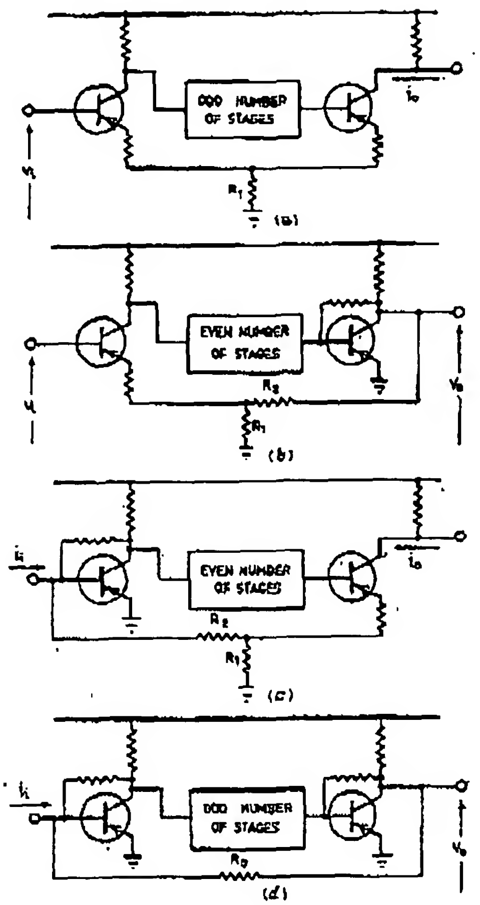


Fig. 13

Elemental mid-band circuit diagrams for amplifiers with overall feedback loops

(a) Stable transfer conductance: $i_o/v_i \simeq 1/R_1$
(b) Stable voltage gain: $v_o/v_i \simeq R_2/R_1$
(c) Stable current gain: $i_o/i_i \simeq R_2/R_1$
(d) Stable transfer resistance: $v_o/i_i \simeq -R_2$

7.3 Theory of low-noise feedback amplifiers

There are several components of noise in the output signal from any amplifier. In the overall-feedback block diagrams of Fig. 13, the noise falls into three main categories:

(a) The inherent noise on the signal.
(b) The noise in the forward path, particularly the input transistor.
(c) The thermal noise due to the feedback resistors.

It is always possible by proper design to reduce the noise contribution of the feedback resistors to any extent, leaving only the irreducible source and input-transistor noise, but unless such care is taken the noise from the feedback resistors can easily be the dominating component.

The noise voltage referred to the input of a series-feedback stage, due to its feedback resistor, is very nearly equal to the open-circuit noise voltage of that resistor, namely

$$v_{nR}^2 = 4kTBR \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Correspondingly, the noise current referred to the input of a shunt-feedback stage, due to its feedback resistor, is approximately the short-circuit noise current of the resistor, namely

$$i_{nR}^2 = 4kTB/R \quad . \quad . \quad . \quad . \quad . \quad (74)$$

For single-stage feedback circuits, the noise contribution of the feedback resistors increases as the loop gain is increased.

The noise voltage at the input due to an overall feedback loop which stabilizes either transfer conductance or voltage gain is the open-circuit noise voltage of the feedback resistor R_1 in Figs. 13(a) and (b). Correspondingly, the noise current from an overall feedback loop which stabilizes current gain or transfer resistance is the short-circuit noise current of R_2 in Figs. 13(c) and (d). This noise decreases as the overall transfer conductance or transfer resistance is increased, provided that the loop gain remains large. The situation is rather more complex in the case of overall loops which stabilize voltage and current gains. In addition, the local feedback resistors contribute to the noise. The noise from the overall feedback resistors is that in the bandwidth which results from closing the overall loop. In contrast, the noise from the local feedback resistors is that in the bandwidth which results from opening the overall loop.

The noise contribution of the overall feedback resistors can be made negligible by suitably choosing their values and by enclosing a sufficient number of stages in the loop to ensure that the loop gain is large. The noise contribution for the local loops can be made negligible also, by making the open-loop bandwidth sufficiently small and using the overall loop to increase the bandwidth to its required value. The most favourable case is when the dominant poles of the forward path are due to the input stage, and they are produced by reducing the local feedback around this stage. For comparison, if a wide-band amplifier simply uses alternate series- and shunt-feedback stages to achieve its bandwidth, the loop gain around all stages must be relatively large and the noise from the feedback resistor around the first stage is likely to be significant.

An example of the use of an overall feedback loop to reduce the noise is the vidicon amplifier described in Section 8.2. In this example a wide-band transfer resistance is synthesized by an overall loop.

8 Two worked design examples

Two complete designs are given in this final Section, to illustrate the practical application of the principles developed in the paper. With one exception, moderately-fast drift transistors, type OC170, are used in both designs.

8.1 A 20 dB 25 Mc/s amplifier for 75 Ω cables

The required specification is a 20 dB amplifier to work between 75 Ω terminations. Two type OC170 transistors are to be used and the bandwidth is to be as great as possible consistent with 5% overshoot. The power supply is 12 V.

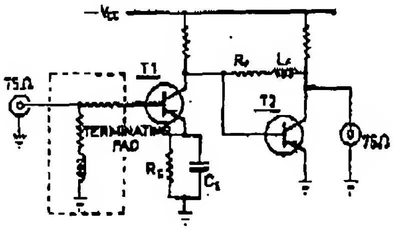


Fig. 14

Elemental circuit diagram of a two-stage amplifier for use with 75 Ω cables

The proposed elemental circuit diagram is shown in Fig. 14. The input transistor is operated at 5 mA emitter current to give τ_T its minimum value of 1 ns. The output transistor available output voltage, but owing to the onset of high-level injection, τ_T rises to about 1·5 ns.

The steps in the design procedure are as follows:

(i) Decide on a 2-pole maximally flat transfer function which has the poles at 45° in the left-half plane. The overshoot will be 4·3%.
(ii) Peak the shunt stage so that its poles are at 45° and there is one zero on the negative real axis.
(iii) Peak the series stage to give one pole on the negative real axis.
(iv) The ratio R_F/R_E will be rather more than 10 (say 12) to give 20 dB gain. Choose the absolute values of R_F and R_E to give cancellation of the axis pole and zero.
(v) Design the input network to give a 75 Ω resistive input impedance.

The complete circuit diagram is shown in Fig. 15. The high-frequency peaking is designed from the equations derived in Section 5.3.1; it is possible to achieve this peaking and be relatively independent of transistor-parameter variation. There is certainly no need to make C_E adjustable, but there is some justification for providing an adjustment to L_F. No adjustments were provided in the prototype.

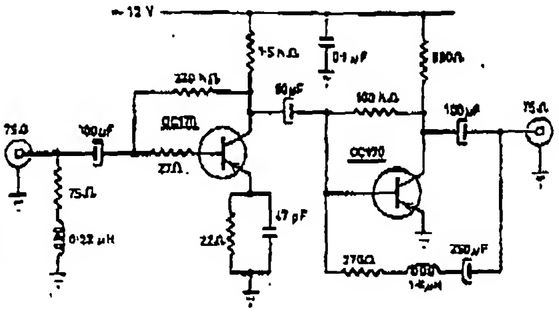

−12 V
7·5 kΩ
0·1 μF
330Ω
220 kΩ
50μF
100 kΩ
100μF
75Ω
75Ω
100μF
OC170
27Ω
75Ω
0·22 μH
22Ω
47 pF
OC170
270Ω
250μF
1·5μH

Fig. 15

Complete circuit diagram of a two-stage amplifier for use with 75 Ω cables

Gain	20 dB
Bandwidth	25 Mc/s
Overshoot	5%

A 75 Ω resistive input termination is achieved by the input pad in conjunction with the input impedance of the transistor. The 27 Ω resistance in the base lead of T_1 gives r_g a nominal value of about 75 Ω; the total input impedance can be made perfectly resistive only if $r_g = Z_0$.

The low-frequency compensation is as follows:

(*a*) Collector feedback biasing is used so that no emitter by-pass capacitors are required. Capacitances of about 10 000 μF would be needed. The price paid is a slight variation of quiescent point with β_N; the 200 and 100 kΩ biasing resistances may require adjustment in the initial setting-up of a unit, but subsequent variations of operating point with temperature or age should not be significant.

(*b*) The blocking capacitance in series with R_F of the shunt-feedback stage is chosen to give approximate low-frequency pole-zero cancellation.

There is negligible tilt on a 50 c/s square wave, and with careful selection of the 250 μF capacitor, a 10 c/s square wave can be reproduced with less than 5% tilt.

Equipment was not available for measuring the high-frequency performance accurately. The fastest equipment available had a rise time of 14 ns and the rise time measured with this equipment was 18 ns with 3% overshoot. This corresponds to a bandwidth of 33 Mc/s, but the possible error is large. It would not be surprising if an accurate measurement showed the bandwidth to be somewhat less than the calculated 27·1 Mc/s, as this frequency is about $\frac{1}{2}\omega_1$, the frequency at which the charge-controlled transistor model becomes quite inaccurate. None the less the performance is quite impressive when it is remembered that the type OC170 is the transistor normally specified for broadcast applications.

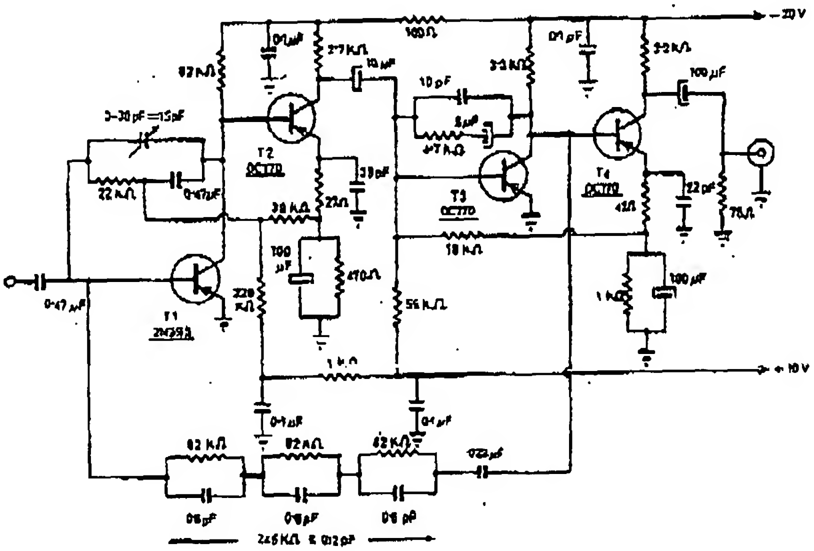

−20 V
100Ω
82 kΩ
0·1 μF
2·7 kΩ
10 μF
0·1 pF
3·3 kΩ
3·3 kΩ
100 μF
10 pF
8 μF
4·7 kΩ
3–30 pF ≈ 15 pF
22 kΩ
0·47 μF
T2
OC170
39 kΩ
22Ω
39 pF
T3
OC170
T4
OC170
47Ω
22 pF
75Ω
18 kΩ
100 μF
470Ω
220 kΩ
56 kΩ
1 kΩ
100 μF
0·47 μF
T1
2N393
1 kΩ
+10 V
0·1 μF
0·1 μF
82 kΩ
82 kΩ
82 kΩ
0·22 μF
0·36 pF
0·36 pF
0·36 pF
246 kΩ ≈ 0·12 pF

Fig. 16

Circuit diagram for a vidicon head-amplifier

Rise time	80 ns
Noise	3·5 nA
Transfer resistance	163 kΩ
Maximum output	250 mV

8.2 A vidicon head-amplifier

Fig. 16 shows the circuit for the vidicon head-amplifier with 5 Mc/s bandwidth. The amplifier has two main sections; the first three transistors with their local and overall feedback loops realize a wide-band low-noise transfer resistance, whilst the final series-feedback stage is used to feed a 75 Ω cable. As stated in Section 7.3, there will be three components of noise at the input. The inherent noise in a vidicon is very small, and so it remains to ensure that the noise due to the feedback resistors is negligible in comparison with that due to the input transistor. The noise current, i_{nT}, referred to the base of a transistor is given by

$$\bar{i}^2_{nT} = 2qI_E\int_0^B\left[\frac{1}{|\beta(f)|^2}+\frac{1}{\beta_N}\right]df \quad . \quad . \quad . \quad (75)$$

Evaluation of the integral and differentiation with respect to I_E shows that minimum noise current is reached when

$$I_{Eopt} = 2\pi\frac{kT}{q}\frac{f_1(c_{tE}+C_S)}{\sqrt{\left[1+\frac{3}{\beta_N}\left(\frac{f_1}{B}\right)^2\right]}} \quad . \quad . \quad . \quad (76)$$

and the minimum noise is

$$\bar{i}^2_{nT\,min} = 8\pi kT\frac{B^3(c_{tE}+C_S)}{3f_1}\left\{1+\sqrt{\left[1+\frac{3}{\beta_N}\left(\frac{f_1}{B}\right)^2\right]}\right\} \quad . \quad (77)$$

where C_S is the stray capacitance. The very best transistors might achieve $i_{nT} = 2 \times 10^{-9}$ A r.m.s. If the noise due to the overall feedback resistor is to be negligible in comparison, it should not exceed 1×10^{-9} A. Eqn. 74 therefore suggests that the overall feedback resistor should be not less than 100 kΩ; the chosen value of 246 kΩ contributes $0{\cdot}6 \times 10^{-9}$ A. In addition, the first transistor has a local shunt-feedback resistor of 22 kΩ. However, with the overall feedback loop open, the bandwidth is only 300 kc/s and the noise contribution of this resistor is therefore $0{\cdot}4 \times 10^{-9}$ A. If the amplifier had been designed as an alternate cascade without the overall feedback loop, the feedback resistor on the input stage would be about 5 kΩ to achieve 5 Mc/s bandwidth and its noise contribution would be 5×10^{-9} A.

The first three stages of the amplifier, with local feedback resistors of 22 kΩ, 27 Ω and 4·7 kΩ, give an open-loop transresistance of 2·6 MΩ, and the closed-loop transresistance is 227 kΩ with a 246 kΩ overall feedback resistor. Fig. 17 shows the high-frequency open-loop singularity pattern for the amplifier; the peaking capacitances were calcul allowing for stray wiring capacitance. The resulting loop pattern was calculated approximately and the tr response was determined accurately with an anal computer.

The input transistor is operated at 150 μA emitter cu for minimum noise. This optimum current is calculated o assumption that $c_{tE} + C_S = 40$ pF when the calculated is $3{\cdot}0 \times 10^{-9}$ A. The input transistor, type 2N393, is no most suitable since it has a rather large value of c_{tE}. T 2N393 is a micro-alloy uniform-base type; a surface-ba transistor would have about half the capacitance and sh achieve 2×10^{-9} A noise.

The biasing system divides the amplifier into two blo each of which consists of a d.c. feedback pair. The input st of each block is a shunt (signal) feedback stage with a l input impedance, so that shunting of the signal by the bia resistors is kept to a minimum; expressed another way, R Fig. 5 is kept as large as possible.[13]

In outline, the low-frequency compensation is as follo

(a) The output stage has a low-frequency pole due to 47 Ω feedback resistor and 100 μF emitter by-pass capacit and a second pole at a much lower frequency due to the outp circuit. The emitter-circuit pole is cancelled by a zero from th first three stages; this zero is due to the 246 kΩ overall fee back resistor and its 0·022 μF series capacitor.

(b) The pole due to the RC coupling network between T and T_3 (2·7 kΩ and 10 μF) is cancelled by a zero from the local feedback loop around T_3 (4·7 kΩ and 5 μF).

(c) The overall loop around T_1, T_2 and T_3 therefore has t main poles, namely 246 kΩ and 0·022 μF due to the overa feedback components, and 27 Ω and 100 μF at the emitter o T_3. A zero is introduced by the local loop around T_1 (22 kΩ

Table 2

PERFORMANCE OF THE VIDICON AMPLIFIER

	Calculation	Measuremen
Transfer resistance into a 75 Ω load	162 kΩ	163 kΩ
Rise time	75 ns*	80 ns
Overshoot	<1%*	<2%
Noise at input	3·0 nA	3·5 nA
Tilt on 50 c/s square wave . . .	—	5%

* The rise time and overshoot were calculated with an analogue computer

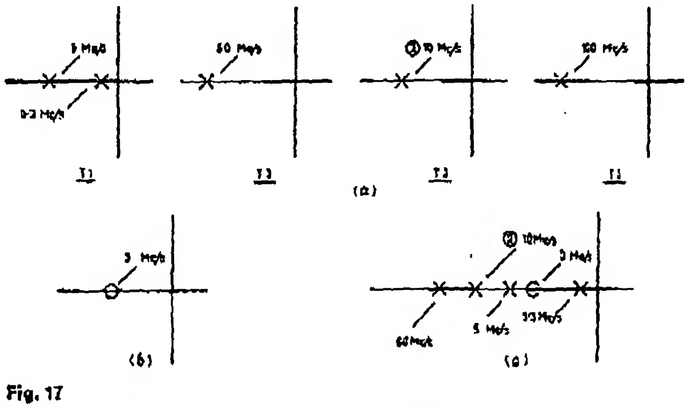


Fig. 17

High-frequency singularity pattern for the vidicon amplifier

(a) Individual-stage poles
(b) Zero due to overall feedback network
(c) Open-loop pattern for overall loop

d 0·47 μF) to reduce the phase shift near the frequency of ity loop gain.

measured and calculated performances are compared able 2. The calculations were made using 'average' nsistor data; unselected production transistors were used the prototype.

Conclusions

It has been shown that the use of alternate series- and unt-feedback stages results in a major simplification in the sign of transistor video amplifiers. The simplification arises imarily because the interaction between stages is both ntrolled and reduced. In addition to the simplification, the uracy of the design is increased and the realizable gain-ndwidth product approaches the ideal limit.

The technique is not restricted to transistor amplifiers and s been successfully applied to valve amplifiers. It is not as nif an improvement with valves as with transistors, ce there is relatively little interaction between valve stages d since the theoretical gain-bandwidth-product limit can be ieved with non-feedback circuits.

Finally, it is worth pointing out that the approach to plifier design outlined in the paper has been developed from oalition of ideas of the two authors, one of whom is ncerned primarily with undergraduate teaching and the er with practical circuit design. This sort of co-operation serve the very useful purpose of closing the gap between academic and empirical approaches to amplifier theory.

10 Acknowledgments

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